

Development Of An Application Through Computational Modeling For The Implementation Of The Riemann Sum Method And Trapezoid Rule To Obtain The Approximation Of Areas Under Curves

Cláudio Faria Lopes Junior¹, Gabriel Soares Da Silva¹, Patrícia Sousa Nilo Mendes¹, Priscila Sousa Nilo Mendes¹, João Henrique Brandenburger Hoppe², Bruno Nunes Myrrha Ribeiro³

¹Department of Metallurgical Engineering, Fluminense Federal University, Brazil

²Department of Mechanical Engineering, Geraldo di Biase University Center, Brazil

³Department of Engineering, Severino Sombra University - Geraldo Di Biase University Center, Brazil

Corresponding Author: Cláudio Faria Lopes Junior

Abstract: Faced with the new challenges encountered by researchers and engineering students, it is increasingly necessary to develop more creative professionals, who have the ability to work using computing as a problem solving tool within their areas. Considering this, this work had the purpose of developing an application through computational modeling for the implementation of the Riemann Sum Method and Trapezoid Rule to obtain the approximation of areas under curves. The methodology used was applied research, consisting of the implementation of the methods in Geogebra software, which generated graphs of the functions inserted through simple programming, based on Significant Learning Theory, to facilitate student learning and understanding the relationship with the theme. In this way, the obtained results were of qualitative character, where the generated application proved efficient to study the methods, due to its illustrative and dynamic way of approaching the subject.

Keywords: Riemann Sum Method, Rule Trapezoid, software Geogebra, theory of Significant Learning.

Date of Submission: 16-07-2018

Date of acceptance: 30-07-2018

I. Introduction

This study had as object of study the calculation of areas under curves by means of numerical methods, specifically using the Riemann Sum method on the right and left, in addition to the Trapezoid Rule. Each of them has its particularity, the approximations are obtained by dividing the area under the curve of the function defined in multiple forms, these being rectangles or trapezoids. The right and left Riemann sum method obtains the approximation with rectangles, where the height of each of them is the value of the function at the end point of its base. When the trapeze is used as a geometric form for the approximation of the areas, it is called the Trapezoid Rule, where it touches the curve at the two upper vertices to obtain the result [1].

The problematic that gave rise to the study in question was the fact that globalization and the pressure coming from companies in search of more creative professionals led the educational institutions to look for new alternatives that facilitate the visualization and the absorption of the content to be transmitted. In view of this, the use of software currently available for educational purposes is becoming essential for the dissemination of knowledge, being more understandable for the entire scientific community. As [2] demonstrated through studies, the possibility of using MATLAB software for teaching in the engineering area, and proving to be effective.

In this way, the objective was to develop an application through computational modeling for the implementation of the Riemann Sum Method and Trapezoid Rule to obtain the approximation of areas under curves. In addition to demonstrating the efficiency of each numerical method, by comparing its results implemented in the program, in an illustrative and dynamic way, thus facilitating the learning and the absorption of the content by the user.

II. Theoretical Perspective

2.1 Numerical Methods

The numerical calculation, defined as a combination of several numerical methods, in which it is an excellent tool to aid in obtaining numerical solutions, in general approximate, of several problems that are found

in daily life. By a numerical method is understood as a set of rules written in the form of a sequence of elementary operations, which can be subtraction, addition, division and multiplication, which lead to a solution of the problem in question [3,4].

With the popularization of low-cost computers and high processing capacity, many of the activities of Science, in particular those of Engineering, have made increasing use of computational methods and techniques in solving real problems, for which manual solutions are impractical, inaccurate, or even very costly in relation to execution time [5].

2.2 Riemann Sum Method

The Riemann sum method is a numerical integration method, whose application is the calculation of areas under curves of a specific function that is represented by $f(x)$, by means of approximations that consists of the division of this area into multiple simple forms, where the amount of the forms used is expressed as n , in this case with rectangle geometry as shown in Figure 1. Since Δx represents the common width of each of the rectangles used and x_{i-1} and x_i their intervals, as expressed in Equation 1-a to sum right and 1-b to left [6].

$$I = \sum_{i=1}^n f(x_i) \Delta x \tag{1-a}$$

$$I = \sum_{i=1}^n f(x_{i-1}) \Delta x \tag{1-b}$$

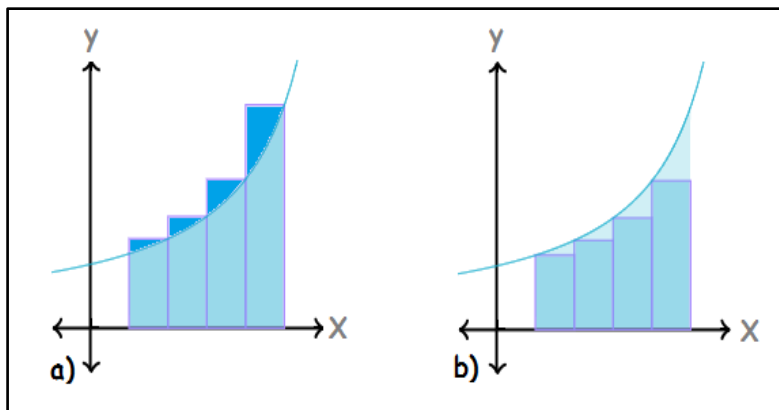


Figure 1: Representation of the Riemann Sum Method (a) to the right and (b) to the left in the area under curve.

2.3 Trapezoid Rule

The approximation of the value of areas under curves is also obtained using instead of rectangles, which are used in the Riemann Sum Method, trapezoids are used in this case to perform numerical integration, known as the Trapezoid Rule, as shown in Figure 2, its geometry is filled in the area under curves according to a stipulated function called $f(x)$. In addition, it is evident that as in other numerical methods, in the Trapezoid Rule the greater the number of segments or forms, expressed as n , the greater its efficiency of approximation of the exact value of the area. Since Δx represents the common width of each of the trapezoids used and x_{i-1} and x_i their intervals, as expressed in Equation 2 [7].

$$I = \sum_{i=1}^n \frac{f(x_{i-1}) + f(x_i)}{2} \Delta x \tag{2}$$

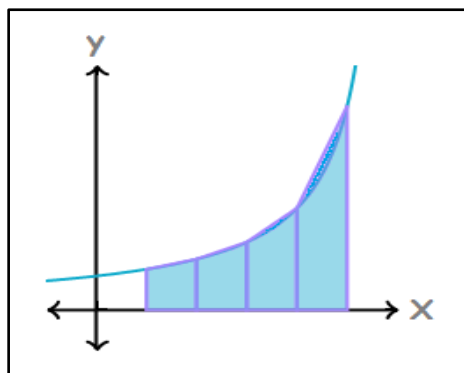


Figure 2: Representation of Trapezoid Rule in the area under curve.

III. Significant Learning Theory

The Significant Learning Theory proposed by David Ausubel, which is gaining more and more space and consolidating itself in the use for Engineering teaching and in other disciplines in general, has as main concept that the previous knowledge of the student is the key to the learning significant amount of new knowledge [8]. Since Ausubel defended and proved by means of his studies that the most important isolated factor and that directly influences the learning is what the learner already knows or dominates, and that the greater the number of links made, the more consolidated the new knowledge will be [9].

In addition, meaningful learning is to broaden and reconfigure existing ideas in the mental structure and thus to be able to relate and access new contents, thus the conceptual maps are used as diagrams that allow the visualization of the relations between concepts approached by the proposed theme, whether both in the classroom and in research on new technologies or tools [10]. As shown in Figure 3, which represents the concepts addressed by this work.

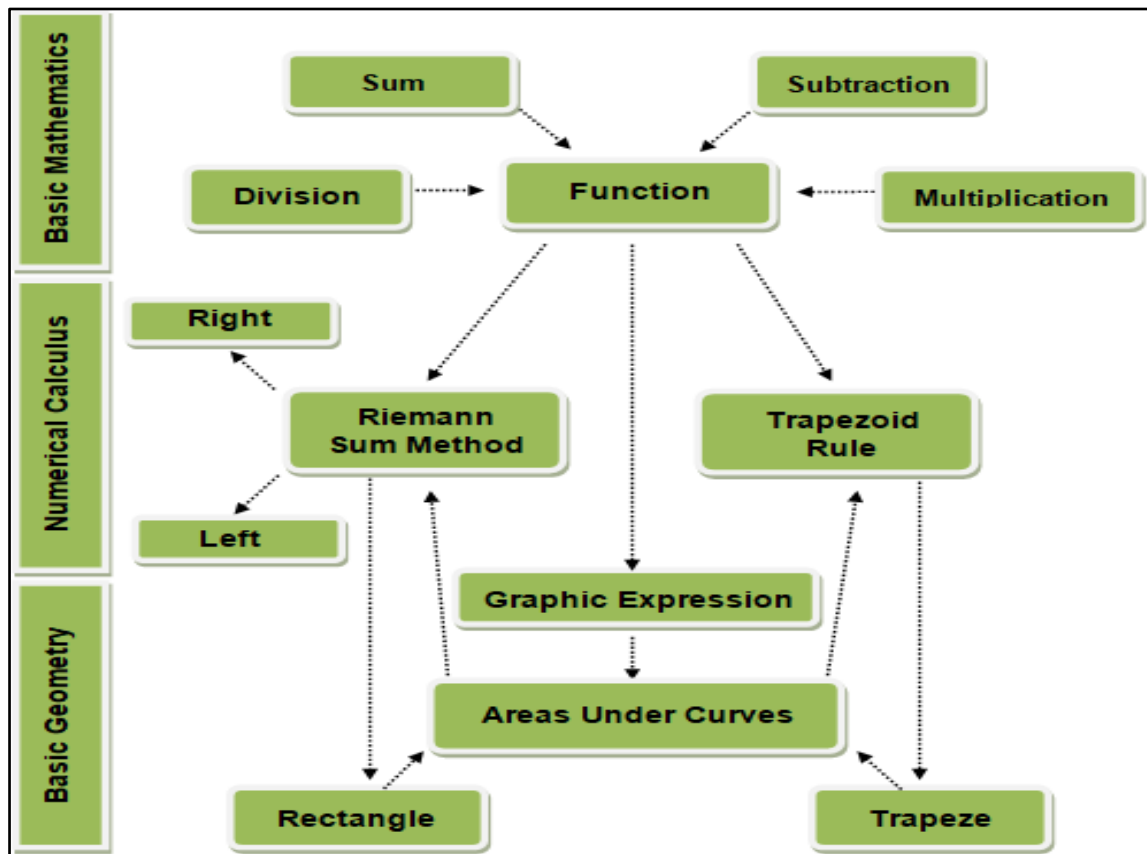


Figure 3: Conceptual Map.

IV. Methodology

4.1. Materials

The following work was developed by the authors in the Laboratory of Modeling and Simulation, located in the interior of the Fluminense Federal University, and it has full access to several softwares that are used by the scientific society of the area, in order to enable the development of studies and research by students and researchers linked to the institution and promote the improvement of Engineering teaching.

Among the available Geogebra software was chosen to develop the study because it is a dynamic mathematics software that has a high efficiency in its practical application, besides having tools that bring together all levels of education, counting on Geometry, Probability, Statistics, Algebra, Calculus Worksheet, Graphs and also Symbolic Calculations, which can be applied to learning in Science, Engineering, Technology and Mathematics. Therefore, the software made possible the implementation of an application that seeks to solve problems of calculations of areas under curves by means of the Riemann Sum Method and Trapezoid Rule, since these can become complex to solve analytically without the aid of computational modeling.

4.2. Methods

The scientific methodology that was used in the study was based on applied research, due to the fact that its purpose is to solve real engineering problems, specifically calculating areas under curves through the implementation of the Riemann Sum Method and Trapezoid Rule. As for the approach, this study was classified as qualitative, since it aimed to improve the quality of teaching related to the theme proposed using Significant Learning Theory, facilitating the understanding of the users through the developed material. In addition, in relation to the technical procedures, the bibliographical research was used to develop the theoretical basis that allows the elaboration of all the work, as well as the experimental research, which represents the entire computational programming process that was carried out, besides the realization of the analysis, which generated the results that were discussed in this study.

Thus, the numerical methods of Riemann Sum and Trapezoid Rule were implemented in Geogebra software, which were adapted to a simple computer programming language to be submitted to the program, by inserting the specific formulations of each method for automatic realization of the calculations of areas under curves proposed by the user, besides the generation of illustrative and dynamic graphs according to the obtained results, representing all the mathematical phenomenon in a way that facilitates the understanding of the same and guarantees a better efficiency in its use.

V. Results and Discussions

5.1 Application

The application proposed by this study resulted in the calculation of areas under curves by the implementation of the Riemann Sum Method and the Trapezoid Rule. In this way, Figure 4 exemplifies the solution of a function, in this case x^3 , with intervals for the calculation ranging from 0.5 to 4, besides having as parameter the number of forms, these rectangles and trapezoids being 15 units for approximation of the real area of the function by implementing the calculation of each method. Thus, it is possible to observe the result of each method for this specific situation.

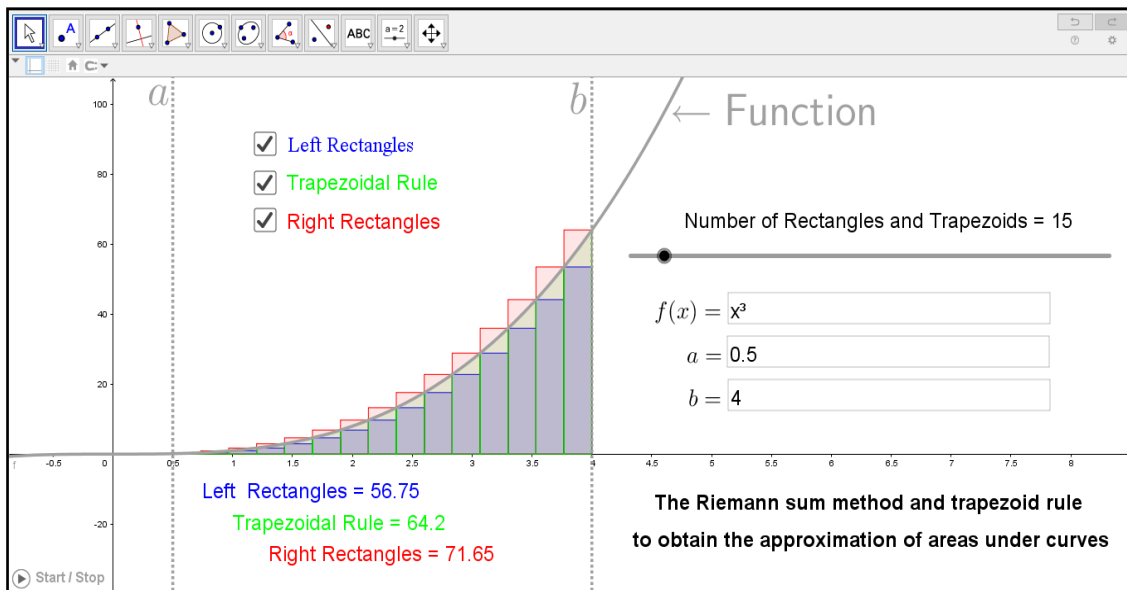


Figure 4: Resolution of the function x^3 by the implementation of the methods in the application.

Since it has the possibility to change the function in which it interferes directly in the graph generated in the program, besides giving the user the possibility of changing parameters, such as the method to be shown, the calculation interval to be executed through the variation of a and b, the number of forms used, being able to vary between rectangles or trapezoids according to the selected method by means of sliders, thus facilitating even more their visualization and interaction, as can be observed in Figure 5.

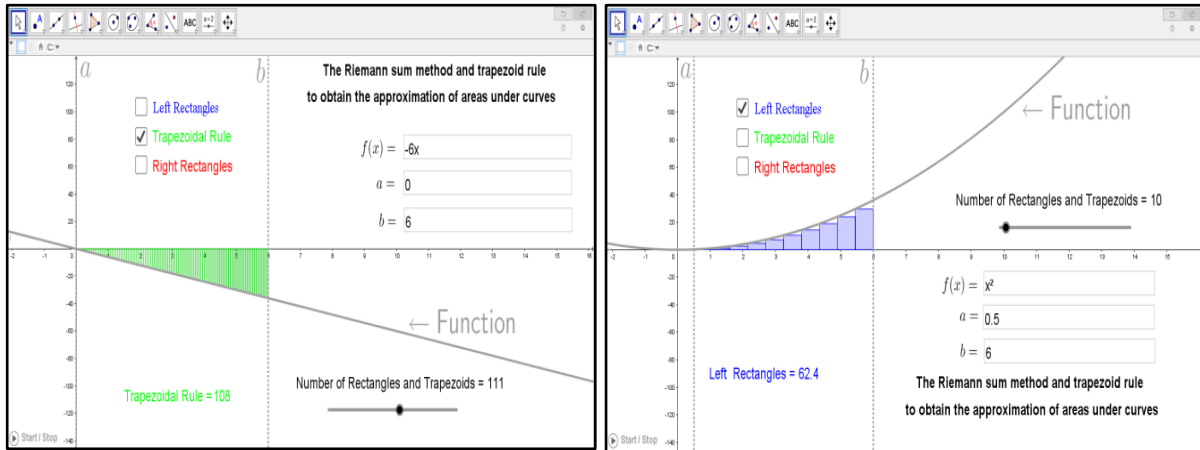


Figure 5: Resolution of the functions $-6x$ and x^2 by the implementation of the methods in the application.

It is important to note that the developed application has the possibility to solve functions in different quadrants, based on simple geometry, when added a negative quadrant with a positive of the same interval, the result would cancel out. And when it comes to the methods used by the program, there are no negative areas for calculus development, so results are added regardless of which quadrant is in relation to the curve, shown in Figure 6.

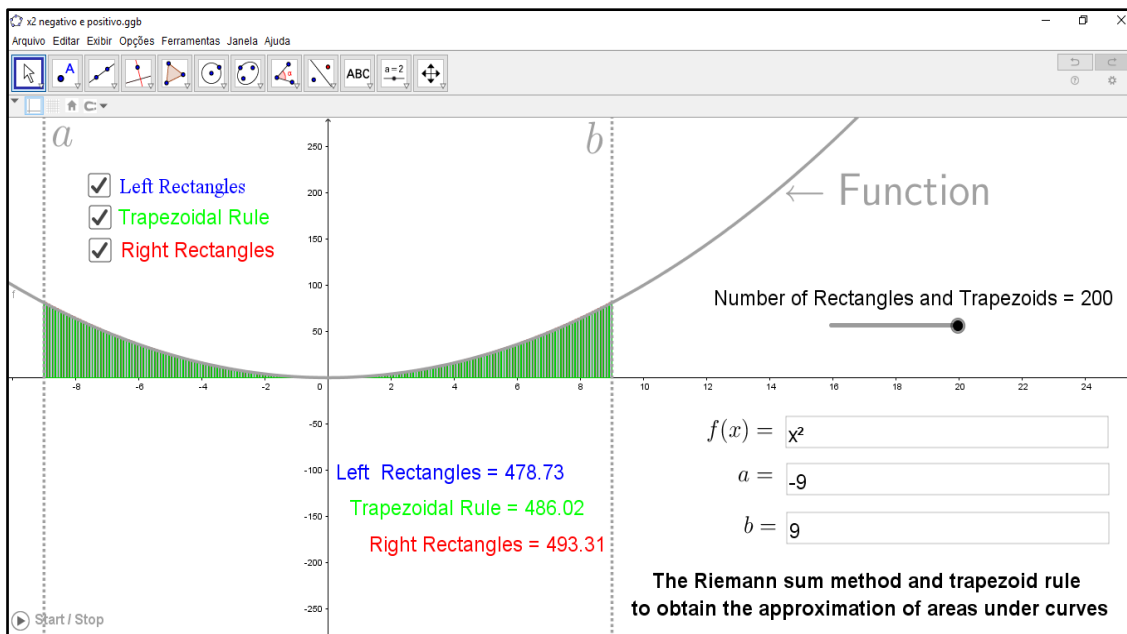


Figure 6: Resolution of the function x^2 in two quadrants by the implementation of the methods in the application.

5.2 Discussions

The study in question has as a discussion about the fact that the covered methods have different efficiencies of their approximations, having as reference the exact result of the curve area, these being justified by the specific forms of each methods, where one is composed of rectangles and another by trapezoids. As can be seen in Figure 7, the Trapezoid Rule proved to be more accurate in its approximation compared to the Riemann Sum Method, since the trapezoid shape is better able to match the curve generated by the determined function. In addition, the greater the number of shapes introduced in the method, the more accurate the result of the method will be.

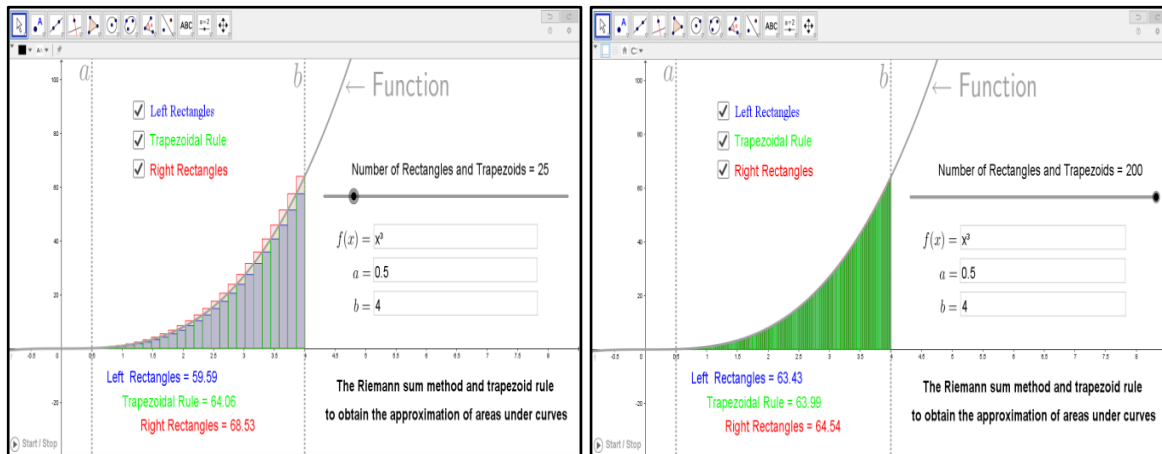


Figure 7: Resolution of the function x^3 with different approximations by the implementation of the methods in the application.

VI. Conclusions

As a conclusion to the following work, the numerical methods of Riemann Sum and Trapezoid Rule are effective for solving Engineering problems, especially in calculations of areas under curves, in which they need to perform operations that are composed of formulations and that are make it complex by their extension or by their need to perform large numbers of repetitions until they approximate the exact value of the proposed problem. Thus, the study shows that, through computational modeling and simulation of problems such as this one, it is possible to improve the efficiency of the obtained results, to increase the quality of the learning of the methods, besides facilitating the understanding of the proposed theme. This leaves open the proposal of possible later work, using theories of learning and the available technologies to increase the quality of teaching and solutions of real problems of the scientific society.

Acknowledgements

The authors of this study would like to thank the Fluminense Federal University for the space of the Modeling and Simulation Laboratory for the development of the project. We would also like to thank the Coordination for the Improvement of Higher Education Personnel – CAPES, for financing the first author in the Master's Degree and the fourth author in the Doctorate. In addition, to Severino Sombra University for the approved Scientific Initiation Project.

References

- [1]. D. V. Griffiths, and I. M. Smith, Numerical Methods for Engineers (Chapman & Hall/CRC: NY, second edition, 2006).
- [2]. C. F. L. Junior, et al, Implementation of numerical methods of Euler and Runge-Kutta through MATLAB software for the Ordinary Differential Equations dedicated to teaching, International Journal of Engineering Research and Application (IJERA), 8(5), 2018, 24-29.
- [3]. A. Greenbaum, and T. P. Chartier, Numerical Methods: Design, Analysis, and Computer Implementation of Algorithms (Princeton University Press: NJ, 2012).
- [4]. S. Butenko, and P. M. Pardalos, Numerical Methods and Optimization: An Introduction (CRC Press: NY, 412 p., 2014).
- [5]. F. Stenger, Handbook of Sinc Numerical Methods (CRC Press: NY, 2016).
- [6]. J. R. Rice, Numerical Methods, Software, and Analysis (Academic Press: NY, 720p., 2014).
- [7]. R. M. L. R. F. Brasil, J. M. Balthasar, and W. Góis, Métodos Numéricos e Computacionais na Prática de Engenharias e Ciências (Editora Blucher: SP, 186p., 2015).
- [8]. D. H. Schunk, Learning Theories: An Educational Perspective (Pearson: NY, 576p., 2011).
- [9]. G. Veletsianos, Emergence and Innovation in Digital Learning: Foundations and Applications (Athabasca University Press: Edmonton, 227 p., 2016).
- [10]. C. F. L. Junior, et al, Uma proposta para o ensino de Resistência dos Materiais com o software Geogebra: Desenvolvimento de um material potencialmente significativo para cálculo de tensão e deformação em carregamentos axiais, Revista Científica Digital da FAETEC, 1(1), 2017, 1-16.

Cláudio Faria Lopes Junior. “Development of an application through computational modeling for the implementation of the Riemann Sum Method and Trapezoid Rule to obtain the approximation of areas under curves” IOSR Journal of Research & Method in Education (IOSR-JRME) , vol. 8, no. 4, 2018, pp. 52-57.